Math 107
Expected Value – The law of Large numbers

Consider the following experiment: Roll a single die and determine whether the number that lands up is even or odd. You know from theoretical probability that P(even) = P(odd) = ½.

Recall the "Law of Large Numbers"

Consider a probability experiment in which the probability of a success in a single trial is a fraction P. Suppose that the single trial of this experiment is repeated many times, and that the outcome of each trial is independent of the others. The larger the number of trials, the more likely it is that the overall fraction of success will be close to the probability P of success in a single trial

Experiment: Roulette

A roulette wheel has 18 black number, 18 red numbers and the numbers 0 and 00 which are green

a) What is the probability of black on any spin?
b) If you spin the wheel 4 times how many blacks do you expect
c) If you spin the wheel 100000 times how many blacks should you expect?

Gambler's fallacy: The **gambler's fallacy**, also known as the **Monte Carlo fallacy** or the **fallacy of the maturity of chances**, is the mistaken belief that, if something happens more frequently than normal during some period, it will happen less frequently in the future, or that, if something happens less frequently than normal during some period, it will happen more frequently in the future (presumably as a means of *balancing* nature). In situations where what is being observed is truly random (i.e., independent trials of a random process), this belief, though appealing to the human mind, is false. This fallacy can arise in many practical situations, but is most strongly associated with gambling, where such mistakes are common among players.

The use of the term *Monte Carlo fallacy* originates from the most famous example of this phenomenon, which occurred in a Monte Carlo Casino in 1913, when the ball fell in black 26 times in a row. This was an extremely uncommon occurrence, although no more or less common than any of the other 67,108,863 sequences of 26 red or black. Gamblers lost millions of francs betting *against* black, reasoning incorrectly that the streak was causing an "imbalance" in the randomness of the wheel, and that it had to be followed by a long streak of red.

Gambler's fallacy – as you play more often your "luck" will change and you will recoup your losses – in the long run in this game – you expect to lose

What is a fair game?

Let's play a game: Roll a die, if a 1 or a 3 comes up, I pay you $2, otherwise you pay me $2. Are you interested?

If we play this game 100 times, what do you expect will happen?

How can we make this game fair?

If a game is fair and we play 100 times, what do you expect will happen?

Expected value:

Expected Value = (value of event 1)x(probability of event 1) + (value of event 2) x (probability of event 2) + …

Expected value gives you your average "winnings" if you play the game for a long period of time

A game is "fair" if the Expected value is 0 (note this different if no money is involved)

Example: Roulette above

You win $1 if you get black, otherwise you lose $1

Example: Insurance:

You sell $100,000 medical insurance claim policy, based on empirical probability that a policyholder will make a claim is 1 out of 500. If they sell a policy $450 each – should they expect to make a profit?

Example: Dice Roll game:

Roll	1	2	3	4	5	6
Payoff	-$1	$2	-$3	$4	-$5	$6

What is the expected value?